Related Rates

1. The volume of a cube grows at a constant rate of $2cm^3/min$.
(a) Compute the rate a side of the cube is growing at the moment the side length is 2 cm.
(b) Compute the rate the surface area is growing at the moment the side length is 2 cm.

2. A candle in the shape of prism with a square base is melting in such a way that its height is decreasing 2 cm/hour, but each side of square cross-section is increasing 0.5 cm/hour. How fast is the volume of the candle changing when the volume is 32 cm^3 and the height is 8 cm?

3. A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.
(a) How fast is the top of the ladder sliding down the wall then?
(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

4. A light is on the top of a 15 ft tall pole and a 6 ft tall person is walking away from the pole at a rate of 2 ft/sec.
(a) At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?
(b) At what rate is the tip of the shadow moving away from the person when the person is 25 ft from the pole?

5. A spotlight is on the ground 20 ft away from a wall and a 6 ft tall person is walking towards the wall at a rate of 2 ft/sec. How fast is the height of the shadow changing when the person is 8 feet from the wall? Is the shadow increasing or decreasing in height at this time?

6. Two people on bikes are horizontally separated by 500 meters. Person A starts riding north at a rate of 1 m/sec and 5 minutes later Person B starts riding south at 2 m/sec. At what rate is the distance separating the two people changing 10 minutes after Person A starts riding?

7. A water trough is 10 m long and a cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the bottom, 80 cm wide at the top, and has height 50 cm. If the trough is being filled with water at the rate of $0.2m^3/min$, how fast is the water level rising when the water is 30 cm deep?

8. Gravel is being dumped from a conveyor belt at a rate of $3m^3/min$, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 3 m high?

9. The minute hand on a watch is 8 mm long and the hour hand is 4mm long. How fast is the distance between the tips of the hands changing at one o'clock?

Extra Problems

1. A ball is being lled with air at a rate of 3 cm3/s. At what rate is the surface area changing when the ball has surface area 36 cm^2?

2. A 10 meter ladder is leaning on a wall and starts to slide down with its bottom end on the ground and top end on the wall. The top of the ladder is sliding down towards the ground at a speed of 2 m/s. Let θ be the angle between the ground and the ladder. At what rate is the angle θ changing when the top of the ladder is 6 meters away from the ground?

3. Suppose that Aylin and Burak are both making cylindrical pots from clay, and both cylinders are 1m tall with a radius of 3 m. Aylin increases only the radius of her pot, whereas Burak increases only the height of his pot, both at the same rate. Which of the two volumes increases faster? (Show your Work.)

4. A particle moves on a hyperbola $x^2 - 18y^2 = 9$ in the first quadrant such that its y-coordinate increases at a constant rate of 9 units per second. How fast is the x-coordinate changing when $x = 9$?

5. One dark night, a policeman running 6 m/s chases a 2 m tall robber running 4 m/s towards a wall. The policeman carries his torch 1m above the ground. At the exact moment when the distance between the two is 30 m, the robber's shadow on the wall is not changing its height. Assuming that they do not change speed, does the policeman catch the robber he climbs the wall?

6. While a 5m tall light source is approaching to a wall by 1 m/s, 2m tall man between the source and a wall is also approaching the source by 1 m/s. When the distance between the light and the man is 6 m, the height of the man's shadow is 1 m. How fast is the height of shadow changing at that moment?